

# STUDY OF FRINGE FIELDS EFFECTS FROM FINAL FOCUS QUADRUPOLES ON BEAM BASED MEASURED QUANTITIES

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## Abstract

Accelerator physics needs advanced modeling and simulation techniques, in particular for beam stability studies. A deeper understanding of the effects of magnetic fields non-linearities will greatly help in the improvement of future colliders design and performance. In [1] and [2], a new tracking method was proposed to study the effect of the longitudinal dependency of the harmonics on the beam dynamics. In this paper, the study will focus on the effects on observable quantities in beam based measurements, for the case of HL-LHC Inner Triplet and with possible tests in LHC.

## INTRODUCTION

In Ref. [1], a new tracking method was presented. It follows the work from Ref. [3] where the magnetic field map or the field harmonics are used to compute first a representation of the vector potential, which enters in the expression of the Hamiltonian, after the non-linear transfer map of the quadrupole is derived using Lie algebra techniques for tracking simulations. A strategy to interface the new map into SixTrack (Ref. [4]) without modifying its internal structure was presented, with some tests, in Ref. [2]. The choice of the integrator's order, z-step size and vector potential's gauge is made in order to optimize tracking accuracy and speed, see Ref. [5] for more details.

Using the new map and the longitudinal magnetic design of the prototype of the Inner Triplet (IT) for HL-LHC (see Fig. 1), this paper studies the impact of a more realistic description of the magnetic harmonic of IT quadrupoles on two observables: the Dynamic aperture (DA) and the amplitude detuning.

The HLLHC V1.0 optics with  $\beta^*=15$  cm (Ref. [6]), and for one configuration (seed 1) of the machine with flat orbit, is used in all simulations unless otherwise specified. The novel method considers non-uniform multipoles distribution along the quadrupole. The IT heads (Fringe Fields) are modelled using 8 different vector potential files, according to the connector side and to the polarity (see Fig. 1), with only the natural harmonic of the quadrupole ( $n=2,6,10,14$ ) for Lie2 ND0 and their derivatives up to order 6 for Lie2 ND6. The central part (body) of the quadrupoles are modelled using thin lenses with integrated multipole kicks, which are computed to keep the total integrated strength for each of the multipoles constant with respect to the other models.

This novel method is compared with uniformly distributed multipoles (called HE model) and uniformly distributed multipoles with additional multipole kicks at the quadrupoles'

extremities (called HE+heads model) as described in Ref. [7]. Random parts of field components (multipole kicks) are also considered in the body of the quadrupole. Finally, to ease the interpretation of the results in the two High Luminosity insertions, only the field errors of the IT are considered.

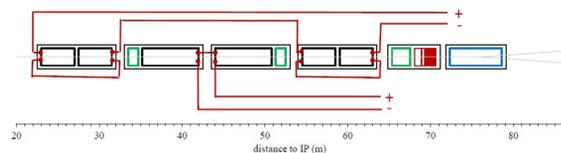
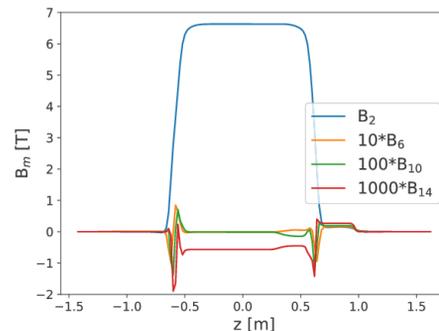


Figure 1: Normal harmonics sampled at  $\Delta z = 2$  mm for the prototype of the IT quadrupole (top). IT powering scheme (bottom). Courtesy of E. Todesco and S. Izquierdo Bermudez

## AMPLITUDE DETUNING

The amplitude detuning studies simulate the particles' motion over  $10^3$  revolutions purely on the vertical or horizontal plane, with and without the dodecapole correction (called MCTX). The initial positions are set to be below the DA value ( $0 < 2J \leq 0.05 \mu\text{m}$ , as shown in next section with a normalized emittance of  $2.5 \mu\text{m}$ ), and their initial momentum offset  $\delta$  is 0. As comparison, the maximum measured amplitude reached in the LHC is of the order of  $0.3 \mu\text{m}$  for a  $\beta^*$  of 25 cm (see Ref. [8–10]).

In a preliminary study, a residual 1<sup>st</sup>-order detuning is observed in all the the models and planes. Part of it is removed by not considering the multipolar errors in the arcs and IR2-8. And when all  $b_4$  multipole components are canceled, the remaining linear detuning is compatible with the 1<sup>st</sup> order anharmonicity given by MADX PTC (Ref. [11]) for the lattice without errors and with the main sextupole. Using a 4<sup>th</sup> order polynomial to fit tracking data, the linear coefficient C1 is about  $1.8 \pm 0.1 \text{e-}2 \mu\text{m}^{-1}$  and  $1.75 \pm 0.1 \text{e-}2 \mu\text{m}^{-1}$ , in the x and y-planes respectively, and is subtracted from the following results. The choice of the polynomial's order for the fit is motivated by using smallest degree for the best score, and it's robustness over fitting procedures.

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Figures 2 and 3 represent the detuning as a function of the initial amplitude for every model. The x-error bars correspond to the minimum and maximum amplitude over the  $10^3$  revolutions, and the y-error bars correspond to the uncertainty of the correction for linear detuning. The detuning from the tracking of each method is compared to the theoretical second order detuning component generated by the  $b_6$  using the formula from Ref. [12]. They agree well up to an amplitude of  $\sim 3.0 \times 10^{-2} \mu\text{m}$ . It is worth noting that this second order is different according to the model, showing that amplitude detuning is an observable sensitive to the longitudinal distribution of the harmonics along the IT quadrupoles. In the horizontal plane and for amplitude higher than  $\sim 3.0 \times 10^{-2} \mu\text{m}$ , a deviation from the second order is visible, whose source is mostly due to the random component of higher order errors.

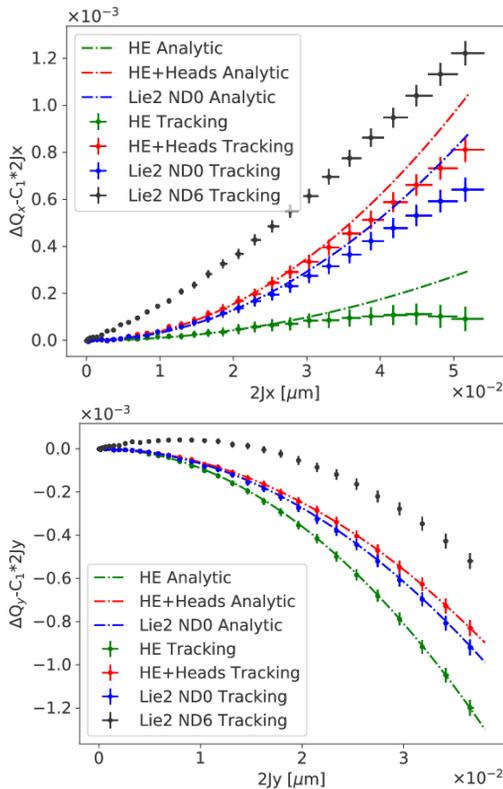


Figure 2: Amplitude detuning for the x-plane (top) and the y-plane (bottom). The  $b_6$  corrector (called MCTX) has been switch OFF.

The comparison between the Lie2 models with and without derivatives (ND0 and ND6, respectively) show an additional linear detuning generated by the  $1^{st}$  and  $2^{nd}$  quadrupole derivatives, as expected from Theory. In a purely horizontal or vertical plane only the  $2^{nd}$  derivatives contribute, but if we look at the coupling term, the contribution of both derivatives are equivalent. The  $3^{rd}$  and  $4^{th}$  derivatives can generate a  $2^{nd}$ -order detuning. Nevertheless, for the Lie2 models, the  $2^{nd}$ -order coefficient from the fit agrees well with a detuning purely due to the  $b_6$  component, as can be seen from Table 1 where the  $1^{st}$  and  $2^{nd}$  order coefficient

of a polynomial fit to tracking data are shown. The linear detuning given by the derivatives (in the Lie2 ND6 model) is of the same order as the one given by the main sextupoles (C1) mentioned previously. The  $2^{nd}$  coefficients are comparable to the theoretical values within their uncertainty for HE and HE with heads models. For the Lie2 models, the theory and the  $2^{nd}$  order detuning are not comparable within the error, and part of discrepancy could be explained by the additional interpolation of the beta-function to have the values at each 2 cm.

Table 1: Amplitude Detuning from Fig. 2 fitted with a  $4^{th}$ -order Polynomial. The Theoretical  $2^{nd}$ -order coefficient is computed using the  $b_6$  component only.

Case	$2\partial Q_x/\partial J_x$	$3\partial^2 Q_x/2!\partial J_x^2$	Theo. $2^{nd}$ order
HE	$(0.6 \pm 0.3)e^{-3}$	$0.08 \pm 0.03$	0.11
HE+Heads	$(0.6 \pm 0.4)e^{-3}$	$0.38 \pm 0.03$	0.39
Lie2 ND0	$(1.4 \pm 0.4)e^{-3}$	$0.22 \pm 0.03$	0.32
Lie2 ND6	$(12.2 \pm 0.4)e^{-3}$	$0.25 \pm 0.03$	0.32

Case	$2\partial Q_y/\partial J_y$	$3\partial^2 Q_y/2!\partial J_y^2$	Theo. $2^{nd}$ order
HE	$(0.2 \pm 0.4)e^{-3}$	$-0.98 \pm 0.05$	-0.90
HE+Heads	$(-0.05 \pm 0.4)e^{-3}$	$-0.62 \pm 0.05$	-0.62
Lie2 ND0	$(0.4 \pm 0.5)e^{-3}$	$-0.79 \pm 0.06$	-0.68
Lie2 ND6	$(10.7 \pm 0.4)e^{-3}$	$-0.67 \pm 0.05$	-0.68

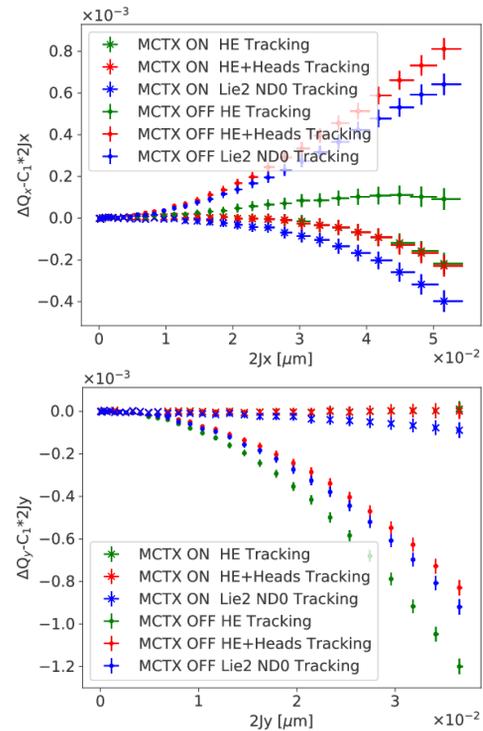


Figure 3: Comparison of amplitude detuning with and without correctors for the  $b_6$  (MCTX).

Figure 3 shows the effect of the corrector for the  $b_6$ , MCTX. In the case of the Lie2 models, its strength is equal

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to the case HE with Heads. Since the case HE and HE with Heads seem to converge, the strength of MCTX appears to be correctly estimated in both cases. However, for Lie2 ND0, the correction is not enough in the vertical plane and too strong in the other.

## DYNAMIC APERTURE

The DA is computed simulating the particles' motion over  $10^4$  revolutions with initial conditions distributed on a polar grid in such a way to have 30 particles (different initial conditions) for each interval of 2 sigma (beam size) from 0 to 28. Five phase space angles, according to field errors, are considered. The initial momentum offset  $\delta$  is set to  $27.e^{-5}$ . The DA values are defined as losses occurred in  $10^4$  turns.

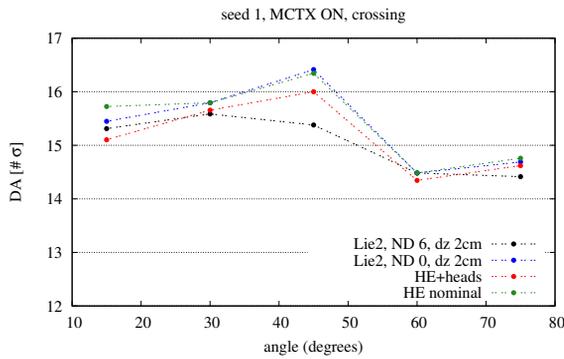


Figure 4: Dynamic aperture for a flat orbit optics, including the corrector for the  $b_6$  (MCTX). The corresponding values for the HE and HE+heads models are shown for comparison.

Figure 4 shows the DA value considering the correction of the  $b_6$  component of the IT (MCTX). The corrector calculation of the HE+heads case is used for the Lie2 model. The different models considered give the same DA within  $1\sigma$ . In particular, the Lie 2 ND6 value at  $45^\circ$  shows the biggest difference with respect to the other models.

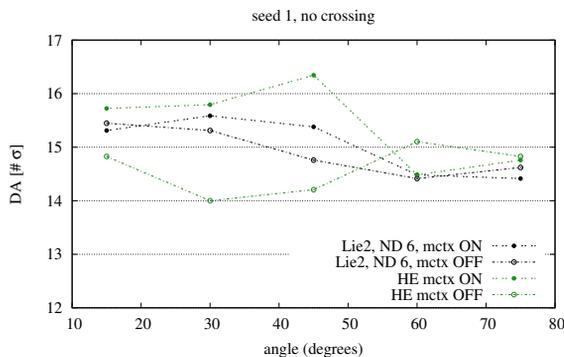


Figure 5: Dynamic aperture comparison with and without  $b_6$  correction (MCTX) for the cases Lie2 ND6 and HE.

When the correction of the  $b_6$  component of the IT is not considered, the DA value still shows the same one  $\sigma$  difference, with a maximum excursion at  $30^\circ$ , as shown in Fig. 5. The inclusion of the derivatives in the field heads is not necessary detrimental for DA though, and the lower DA

at  $45^\circ$ , when the  $b_6$  correction is considered, seems to be due to the ineffectiveness of the correction, computed in the case HE+heads, for the case Lie2 ND6.

The minimum DA and the average DA values over 60 seeds are reported in Table 2 for each angle scanned in the tracking simulations. In particular, the minimum values are comparable for all the models within one  $\sigma$ , and the average values are equivalent between the different models (with a difference below  $0.5\sigma$ , which is the DA calculation precision). The DA at  $10^5$  revolution has been calculated for the HE and the Lie2 ND6 models only. All the values are well within  $0.5\sigma$  between the two models, i.e. converge to the same DA within its precision.

Table 2: Minimum and Average Computed DA Over 60 Seeds and  $10^4$  Turns, for Each Angle and Model Considered

Case	15°	30°	45°	60°	75°
minimum DA					
HE	14.8	15.0	15.5	13.8	13.9
HE+Heads	14.6	15.0	15.5	13.6	13.7
Lie2 ND0	14.7	15.0	15.6	13.4	13.7
Lie2 ND6	14.4	15.0	14.7	13.1	13.6
average DA					
HE	15.6	15.8	16.2	15.0	14.7
HE+Heads	15.5	15.6	16.2	14.7	14.5
Lie2 ND0	15.5	15.8	16.5	14.8	14.6
Lie2 ND6	15.4	15.6	16.1	14.5	14.3

## CONCLUSION

The possibility to study Fringe Fields effects has been added to SixTrack code (Ref. [13]). The impact on beam based observables are studied in this paper, using tracking simulations of the HLLHCv1.0 optics.

The computation of DA as losses at  $10^4$  or  $10^5$  turns is not so sensitive to the longitudinal distribution of the field. The time evolution of the DA will be better investigated for the different models.

On the contrary, amplitude detuning is a good observable to see those effects. In this respect, beam based measurements on LHC show about 30% more linear amplitude detuning than expected by integrated magnetic measurements in the LHC (Ref. [10]). Since amplitude detuning is sensitive to Fringe Fields, it is planned to study their impact on the secondary lines of the frequency spectrum (RDTs) and on the coupling terms of the amplitude detuning.

The possibility to use  $\beta$ -beating or the phase advance as observables has also been attempted, using Machine Development data of 2016, 2017 and 2018 (Ref. [8–10]). No clear dependence with the amplitude has been observed so far.

## ACKNOWLEDGEMENT

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