STUDY OF THE SYSTEM STABILITY FOR THE DIGITAL LOW LEVEL RF SYSTEM OPERATED AT HIGH BEAM CURRENTS


Abstract

The purpose of a Low-Level Radio Frequency (LLRF) system is to control the amplitude and phase of the field in the accelerating cavity. A digital LLRF (DLLRF) system will be installed in the Taiwan Photon Source (TPS) storage ring in 2019. The system stability depends much on the feedback parameters. An instability of the cavity voltage controlled by a DLLRF was observed during machine tests with high beam current and low feedback gain. A simulation model for the digital LLRF system with beam-cavity interaction was developed to investigate this instability and simulations and machine test results will be presented here.

INTRODUCTION

The Taiwan Photon Source (TPS) at NSRRC is a third-generation light source operating at 3 GeV electron energy. To have better RF field stability, precise control and high noise reduction, a digital Low-Level Radio Frequency (DLLRF) control system based on Field Programmable Gate Arrays (FPGA) was developed at the NSRRC [1, 2]. We replaced the analogue LLRF system with a digital version for the TPS booster ring at the beginning of 2018 and enjoyed a stable and successful operation [3, 4]. The DLLRF systems for the TPS storage ring are under testing and will be installed in the near future. The performance of the DLLRF during machine tests for the TPS storage ring can be found in [5].

During machine tests with high beam current, a cavity voltage instability was observed. Figure 1 shows such an unstable event at 390 mA beam current and with two SRF modules operating at 1600 kV cavity voltage. A 3.4 kHz oscillation of the cavity voltage occurred in both SRF modules and the RF systems were tripped by the interlock protection system. This instability can limit the maximum storable beam current and may result from beam-cavity-LLRF interactions. A simulation model for the digital LLRF system was developed to investigate this instability and its method and results are discussed in the following sections.

SIMULATION MODEL

A simple LLRF system model was developed with MATLAB and Figure 2 shows its block diagram. Each RF station has its own controller and plant models. The RF plant model includes the cavity response and the calibration of the DAC to generator current and the cavity voltage to ADC. The beam model receives cavity voltage information from each RF stations and passes the beam phase information back to each cavity model. The amplitude of the beam current is assumed to be constant.

![Figure 1: An unstable event at 390 mA beam current during machine test.](image1)

![Figure 2: Block diagram of the DLLRF control model with beam-cavity interaction.](image2)

The equations of the cavity model including the beam current can be written as follows [6]:

\[ V_{ct} + \omega_{1/2} V_{ct} + \Delta \omega V_{cq} = \omega_{1/2} R_{t} (I_{ct} + I_{b}) \]  

(1)

\[ V_{cq} + \omega_{1/2} V_{ct} - \Delta \omega V_{ct} = \omega_{1/2} R_{t} (I_{c0} + I_{b}) \]  

(2)

where \( \omega_{1/2} = \omega_{cav}/2Q_{L} \) and \( \Delta \omega = \omega_{cav} - \omega_{RF} \) and where the calibration coefficients \( G \) and \( \Delta \) for \( I_{b} = G \cdot DAC \) and \( ADC = A \cdot V_{c} \) are obtained from measurements. Figure 3 shows the fitting results of the calibration coefficients for RF station #3. The coefficient \( G \) can be calculated from the following relations:

\[ \sqrt{P_{f}} = a_{0} \cdot DAC \]  

(3)

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For the present design of the PI control algorithm, we use the theoretical form including a 13-bit truncation in the digital implementation. The equations of the controller model can be defined as follows:

\[ I_c = \sqrt{P_I \cdot \frac{8\beta}{(1 + \beta) \cdot R_L}} \]  

\[ G = a_0 \cdot \sqrt{\frac{8\beta}{(1 + \beta) \cdot R_L}} \]

For the present design of the PI control algorithm, we use the theoretical form including a 13-bit truncation in the digital implementation. The equations of the controller model can be defined as follows:

\[ e[k] = r[k] - y[k] \]  

\[ u[k] = \frac{1}{2^{13}} K_p \cdot e[k] + \frac{1}{2^{13}} \sum_{i=1}^{13} K_i \cdot e[i] \]

where \( K_p \) is the proportional gain, \( K_i \) the integral gain, \( y \) the input ADC for the cavity voltage and \( u \) the output DAC.

The beam phase can be obtained from the Pedersen model, which is commonly used to study the interaction between control and RF system [7, 8]. The differential equations of the beam phase can be written as follows:

\[ \dot{\phi}_b = 2\pi \alpha \frac{eV_{0}}{E T_0} \sin \phi \left( \phi_t - \phi_s + \pi \right) - 2\alpha \phi_s \]  

\[ \phi_s = \phi_{0} - \frac{(V_{\text{cav}} - V_{0})}{V_{0}} \tan \phi_s \]

where \( h \) is the harmonic number, \( \alpha \) the momentum compaction factor, \( E \) the electron beam energy, \( T_0 \) the revolution time, \( \phi_{0} \) the synchrotron oscillation damping term, \( V_{\text{cav}} \) the cavity voltage amplitude and \( V_{0} \) the desired cavity voltage amplitude. The parameters for the TPS storage ring and its RF system are given in Table I.

### Table 1: Parameters of the TPS and Its RF System

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency, ( f_{\text{RF}} )</td>
<td>499.65 [MHz]</td>
</tr>
<tr>
<td>Beam Energy, ( E )</td>
<td>3 [GeV]</td>
</tr>
<tr>
<td>Loaded quality factor, ( Q_L )</td>
<td>6.6×10^4</td>
</tr>
<tr>
<td>Unloaded quality factor, ( Q_0 )</td>
<td>1.8×10^9</td>
</tr>
<tr>
<td>Shunt impedance, ( R_s )</td>
<td>8.55×10^{10} [Ω]</td>
</tr>
<tr>
<td>Momentum compaction factor, ( \alpha )</td>
<td>2.4×10^{-4}</td>
</tr>
<tr>
<td>Radiation loss/turn, ( U_0 )</td>
<td>853 [keV]</td>
</tr>
<tr>
<td>Harmonic number, ( h )</td>
<td>864</td>
</tr>
</tbody>
</table>

### RESULTS AND DISCUSSION

To study system stability with feedback, the RF system is assumed in the simulation to be initially in a steady state without feedback followed after some time by activation of the feedback loops. The time duration for the simulation is set to 180 msec. If the cavity voltage increases with time while the feedback loops are activated or the deviation of the cavity voltage is larger than 20 kV, the system is considered to be unstable.

Figure 4 shows one of the simulation results. The cavity voltage has a 3.3 kHz oscillation at 390 mA beam current with \( K_p = 6000 \) and \( K_i = 80 \). This result is consistent with observations shown in Figure 1.

Figure 4: A simulated unstable event at 390 mA with 3.3 kHz cavity voltage oscillations.

To investigate the effects of operational parameters on system stability, we use a simplified simulation which only contains one RF station. The most important operational parameters for the DLLRF are the PI gains \( K_p \) and \( K_i \). Figure 5 shows how the parameters \( K_p \) and \( K_i \) affect the maximum stored beam current.

Figure 5: Maximum stored beam current as a function of PI gains. The Robinson stability limit is also shown.

The maximum stored beam current is mostly affected by \( K_i \), which corresponds to the system bandwidth. The maximum stored beam current decreases with increasing values for \( K_i < 10 \), followed by an increase at larger \( K_i \) and finally saturating for \( K_i \) larger than 55. The decreasing maximum storable beam current in the low \( K_i \) region may be due to...
influences of the system bandwidth and synchrotron frequency, which was also observed at the analogue LLRF system [9, 10]. The increasing maximum stored beam current in the high $Ki$ region may be due to the limited response time of the feedback system to sustain stability. The limit of the maximum stored beam current at $Ki > 55$ may come from the system time delay, which means the short response time may cause the feedback system to overreact and render the system unstable.

The calibration coefficients $A$ and $G$ also affect the system stability and can be adjusted by inserting the RF amplifier or attenuator in the RF path. Figure 6 shows how the calibration coefficients $A$ and $G$ affect the maximum stored beam current. The larger the calibration coefficients $A$ and $G$ are, the higher the maximum storage beam current will be. However, the $A$ and $G$ are limited by the resolution of the DAC and ADC, where the optimized calibration coefficients do not give the highest maximum stored beam current. In the TPS case, the maximum output RF power is about 290 kW and the maximum allowed cavity voltage about 2400 kV for each RF station operated in the RF processing mode. The DAC and ADC are both in the range of ±8192. Therefore, to avoid saturation, reach maximum stored beam currents and enough resolution, the calibration coefficients for $A$ and $G$ are selected to be about 7200 ADC counts in amplitude for 2400 kV of cavity voltage and 6400 DAC counts in amplitude for 290 kW of RF output power.

![Figure 6: Maximum stored beam currents as a function of calibration coefficients.](image)

**CONCLUSION**

A DLLRF control system was developed and is currently under testing in the TPS storage ring. During machine tests with high beam currents, an instability of the cavity voltage was observed. A simple LLRF system model, including beam-cavity interaction, was developed to study the system instability at high beam currents. This model predicts the cavity voltage oscillation under certain operational conditions and is consistent with observation. The most important operational parameters for system stability are the $Ki$ of the DLLRF control system. The calibration coefficients for the ADC and DAC should be set properly as well. The operational parameters for the TPS DLLRF control system were chosen by the results of this model and up to 400 mA no unstable events were observed anymore with optimized $Kp$, $Ki$ and calibration coefficients.

**REFERENCES**


