First Experimental Measurements of the Caustic Nature of Trajectories in Bunch Compressors

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The envelope of trajectories forms the caustics lines.
Caustics lines are an envelope of trajectories, or the projection of that envelope on another surface.

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Outline

1. Introduction of caustics
2. Example of the usefulness of caustics
3. MAX IV experimental set up & results
Caustics in accelerator physics

Electron trajectories in a bunch compressor:

Where there are caustics, there will be current spikes.

Optical caustics:
Caustics are singularities in the density of families of trajectories.
Caustics fall into catastrophe theory

Figure: EC Zeeman (1976) Catastrophe Theory in Scientific American.
Phase space evolution through a bunch compression
Phase space evolution through a bunch compression

Energy spread induced by CSR:

\[
\frac{dE}{cdt} = \frac{-2e^2}{4\pi\epsilon_0(3R^2)^{1/3}} \int_{\tilde{z} - z_L}^{\tilde{z}} \frac{d\lambda}{dz} \left( \frac{1}{\tilde{z} - z} \right)^{1/3} dz
\]

\[\lambda = \text{linear charge density}\]

Current spikes are problematic, leading to greater **CSR-induced emittance growth.**
Caustic expression

\[ \tilde{z}_c(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)} \]

\[ \tilde{R}_{56}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)} \]


\[ \delta(z_i) = h_1 z_i + h_2 z_i^2 + h_3 z_i^3 \]
Caustic expression for a chicane

\[ R_{56} = -10 \text{ mm} \]
Fold

$T_{566} = 18.1 \text{ mm}, U_{5666} = -24 \text{ mm}$

E.g. single spike bunch compression

Cusp

$T_{566} = 16.4 \text{ mm}, U_{5666} = -11.4 \text{ mm}$

E.g. LCLS
[Y. Ding et al. Phys. Rev. AB 19, 100703]

Butterfly

$T_{566} = 16.1 \text{ mm}, U_{5666} = 2.6 \text{ m}$

E.g. MAX IV
[S. Thorin et al. FEL2010, WEPB34]
Example of caustics

Current horn suppression
Current horn suppression
An S-band FEL Linac example

Elegant simulations of longitudinal phase space and current profile at the end of Linac2.
Caustic condition

\[ R_{566} \frac{d(\delta(z))}{dz} \bigg|_{z=z_i} + T_{566} s \frac{d}{dz} \left( \delta^2(z) \right) \bigg|_{z=z_i} + U_{5666} s \frac{d}{dz} \left( \delta^3(z) \right) \bigg|_{z=z_i} + s_{BC} = 0. \]

where,
\( \delta = \) energy spread
\( z_i = \) initial longitudinal position
\( s = \) position along accelerator
\( s_{BC} = \) end of bunch compressor position
Caustic condition

\[ R_{566} \frac{d(\delta(z))}{dz} \bigg|_{z = z_i} + T_{566} \frac{d}{dz} \left( \frac{d^2(\delta(z))}{dz^2} \right) \bigg|_{z = z_i} + U_{5666} \frac{d}{dz} \left( \frac{d^3(\delta(z))}{dz^3} \right) \bigg|_{z = z_i} + s_{BC} = 0. \]

where,
\( \delta = \) energy spread
\( z_i = \) initial longitudinal position
\( s = \) position along accelerator
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Caustic current horn suppression
S-band FEL linac example

Octupole added to BC1 to vary U5666.
Caustic current horn suppression

Without octupole:

With octupole:

63% reduction in the CSR-induced emittance growth.

$\epsilon_{nx} = 1.359 \text{ mm mrad}$

$\epsilon_{nx} = 0.762 \text{ mm mrad}$

Experimental results from MAX IV
MAX IV

Electron guns

Linac

1.5 GeV ring

3 GeV ring

Linac

1.5 GeV

3 GeV

SPF
MAX IV Linac

- Photocathode and thermionic electron guns
- 2 double achromat bunch compressors (BCs)
- RF frequency 2.998 GHz
MAX IV Bunch Compressors

Double achromat bunch compressors:
• positive fixed R56 (energy chirp from falling RF slope)
• natural T566 used for linearization, self-linearizing
• weak sextupoles at the center of each achromat used for fine tuning T566
Zero-crossing method

Sextupole strengths varied

RF phase at 40°

Extraction 1.5 GeV ring
Extraction 3 GeV ring

Screen (after first dipole)
Zero-crossing method

RF phase at 40°

Extraction 1.5 GeV ring
Extraction 3 GeV ring

Sextupole strengths varied

PG
LG
TG
BC1 @ 260 MeV
BC2 @ 3 GeV

SPF

Intensity (a.u.)

x (pixels)
Experimental results – linearly ramped current profiles

Sextupole strengths [m^{-3}]:

1.0  7.0  13.0  19.0  25.0  31.0  37.0

![Graphs showing intensity vs. position for different sextupole strengths.](image-url)
The origin is found as being the distance from one bunch edge to the position within the bunch where the accumulated integrated charge equals half of the total charge.

**Caustic expression:**

\[
 z_c(z_i) = -z_i - \frac{R_{56} \delta(z_i)}{2} + \frac{U_{5666} \delta^3(z_i)}{2} + \frac{\delta(z_i)}{2\delta'(z_i)} \\
\tilde{T}_{566}(z_i) = \frac{1}{2\delta(z_i)} \left( -R_{56} - \frac{1}{\delta'(z_i)} - 3U_{5666} \delta^2(z_i) \right)
\]

(Same expression as shown on slide 11, simply re-arranged)
Mapping out caustic lines

Offset due to the combined effects of wakefields and curvature of the phase space distribution.
Mapping out caustic lines

Where the caustic expression is undefined, the longitudinal phase space is considered well linearized, resulting in a short bunch with a symmetrical current profile.

\[
\begin{align*}
    z_c(z_i) &= -z_i - \frac{R_{56}\delta(z_i)}{2} + \frac{U_{5666}\delta^3(z_i)}{2} + \frac{\delta(z_i)}{2\delta'(z_i)} \\
    \tilde{T}_{566}(z_i) &= \frac{1}{2\delta(z_i)} \left( -R_{56} - \frac{1}{\delta'(z_i)} - 3U_{5666} \delta^2(z_i) \right)
\end{align*}
\]

Offset of 14 micrometers removed
Conclusions

- Caustics are a useful way that we can look at various types of focusing.
- Demonstration of current horn suppression through manipulation of the higher-order dispersion.
- Preliminary experimental results show good agreement in shape with the theory.
- In addition to validating the caustic expressions, these measurements demonstrate the effectiveness of optical linearization of the MAX IV bunch compressors.
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